

# On the Casimir energy of the electromagnetic field in the dispersive and absorptive medium

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## Abstract.

The microscopic theory of the Casimir effect in the dielectric is studied in the framework when absorption is realized via a reservoir modeled by a set of oscillators with continuously distributed frequencies with the aim to see if the effects depend on the form of interaction with the reservoir. A simple case of the one-dimensional dielectric between two metallic plates is considered. Two possible models are investigated, the direct interaction of the electromagnetic field with the reservoir and indirect interaction via an intermediate oscillator imitating the atom. It is found that with the same dielectric constant the Casimir effect is different in these two cases, which implies that in the second model it cannot be entirely expressed via the dielectric constant as in the well-known Lifshitz formula.

## 1 Introduction

The consistent quantum-mechanical treatment of the Casimir forces in the dispersive and absorbing dielectric requires inclusion of the absorbing medium as an independent dynamical system. This problem has attracted attention since long ago. A formalism allowing to consider absorption of the electromagnetic field in the medium in the microscopic approach was developed by B.Huttner and S.M.Barnett [1]. It consists in modeling the medium as a set of oscillators with continuously distributed frequencies. Interaction with the medium leads to absorption of the electromagnetic field and its exclusion as an independent dynamical variable. The resulting system is completely described by a set of effective oscillators, which also have continuously distributed frequencies. Quantum field excitations are expressed via the ones of these effective oscillators ("polaritons").

Within this or similar picture introduced explicitly or assumed implicitly derivation of the Casimir energy can be done using the macroscopic expression for it and interpreting the electromagnetic field as a quantum operator satisfying the Heisenberg equations in which the influence of the medium appears as a "quantum noise" leading to absorption. Taking the average in the ground state one obtains the Casimir energy of the field in the presence of the medium. This or similar approach was presented in various publications [2, 3, 4, 5]. The problem of such a treatment is taking fully into account the interaction energy between the field and medium and determination of the ground state, which may change with this interaction.

The consistent treatment requires to determine the ground state of the total Hamiltonian and take the average in this state. Such a program was accomplished in [6, 7], where however a simplified version of the original Huttner-Barnett (HB) model was studied. In the HB model the electromagnetic field is assumed to interact directly only with atoms in the dielectric and

simplified version the field directly interacts with the medium without the intermediary atom (the direct (D) model). Additionally, to avoid the change of the ground state with interaction, the authors of [6] chose the interaction in an unnatural way to depend separately on creation and annihilation operators for the field and medium variables. The final formulas of both calculations are somewhat different. They both have the form of the classical expression for the energy in which the (real) refractive index  $n = \sqrt{\epsilon}$ , where  $\epsilon$  is the dielectric constant, is to be substituted by the full complex  $n$  in [7] and by its real part in [6].

With all this the Casimir energy in the initial HB model has never been calculated consistently, that is as the ground state energy of the total Hamiltonian taking into account that the ground state itself changes with interaction. This calculation occupies the main part of this paper.

Note that this problem is of a wider scope. Modeling the absorbing medium and its interaction with the field it is important to know if the final results for, say, the Casimir energy depend on different properties of a particular model or this dependence is wholly concentrated in the way the field propagates in the medium, that is in the complex and frequency-dependent electromagnetic constants  $\epsilon$  and  $\mu$  of the medium. A remarkable result found in papers [6] and [7] (although different) is that the Casimir energy can be expressed entirely in terms of the frequency dependent complex dielectric and magnetoelectric constants thus linking this approach with the standard macroscopic Lifshitz formula [8]. The influence of the medium was found to be implicitly included into the properties of the two constants. The question is whether this result is general or restricted to specific forms of the model.

In this paper we show that for models in which the field interacts directly with the medium, which are generalizations of the simple models of [6] and [7], this result remains valid. However for the more complicated original HB model it does not. Due to complexity of the derivation in the HB model we obtained this result only for a particular simple case of one dimensional electromagnetic field between two metallic plates. Moreover even with this simplification we could find it only numerically and with a specific form of the atom-medium interaction. With this interaction in the HB model we found both the dielectric constant  $\epsilon(\omega)$  and Casimir energy. Then we compared this energy with the one calculated in the simple model of T.Philbin [7] using the same  $\epsilon(\omega)$ . If the energy is wholly determined by the dielectric constant the result should also be the same. However in fact the resulting Casimir energy proved to be very different indicating that the Casimir energy depends not only on  $\epsilon(\omega)$  but on the details of the model. So the answer to the question whether the Casimir force is uniquely determined by the dielectric (and in all probability magnetoelectric) constant seems to be negative.

This also means that at least in this framework the experimental study of the Casimir force may give some information about the dynamical mechanism behind dispersion and absorption in the medium.

## 2 Simple model (D model)

We start with a simple model introduced by T.Philbin for the quantization of the electromagnetic field in the dispersive and absorptive medium. To further simplify we restrict ourselves with a homogenous dielectric as the medium, represented microscopically by field  $\mathbf{Y}_\omega$  with continuously distributed frequencies. The Lagrangian density splits into three parts  $\mathcal{L} = \mathcal{L}_e + \mathcal{L}_r + \mathcal{L}_i$ . Here  $\mathcal{L}_e$  is the Lagrangian density of the electromagnetic field

$$\mathcal{L}_e = \frac{1}{2}(\dot{\mathbf{E}}^2 - \mathbf{B}^2), \quad (1)$$

$\mathcal{L}_r$  is the Lagrangian density of the "reservoir",

$$\mathcal{L}_r = \frac{1}{2} \int_0^\infty d\omega \left( \dot{\mathbf{Y}}_\omega^2 - \omega^2 \mathbf{X}^2 \right) \quad (3)$$

and finally  $\mathcal{L}_i$  is the Lagrangian density for the interaction, which we take following [7]

$$\mathcal{L}_i = - \int_0^\infty d\omega v(\omega) \mathbf{A} \cdot \dot{\mathbf{Y}}_\omega, \quad (4)$$

where  $v^2(\omega)$  is a square integrable function which can be analitically continued to negative  $\omega$  as an even function. In contrast with T.Philbin we do not introduce interaction with the scalar potential  $\phi$  nor with the magnetic field assuming that the medium is magnetically neutral. In the Coulomb gauge the dynamical part of the Lagrangian density becomes expressed effectively via transverse fields

$$\mathcal{L}_\perp = \frac{1}{2} (\dot{\mathbf{A}}^2 - \mathbf{B}^2) + \frac{1}{2} \int_0^\infty d\omega \left( \dot{\mathbf{Y}}_{\omega\perp}^2 - \omega^2 \mathbf{Y}_{\omega\perp}^2 \right) - \int_0^\infty d\omega v(\omega) \mathbf{A} \cdot \dot{\mathbf{Y}}_{\omega\perp}. \quad (5)$$

Passing to the momentum space the transverse Lagrangian becomes a sum over two polarizations  $\lambda = 1, 2$  with

$$L_\lambda = \int d^3k \left\{ \frac{1}{2} \left( |\dot{A}_\lambda|^2 - k^2 |A_\lambda|^2 \right) + \int_0^\infty d\omega |\dot{Y}_{\omega,\lambda}|^2 - \omega^2 |Y_{\omega,\lambda}|^2 \right\} - \int_0^\infty d\omega v(\omega) X_\lambda^* \dot{Y}_{\omega,\lambda}. \quad (6)$$

Each polarization is treated similarly and in following subindex  $\lambda$  will be suppressed.

At this point we introduce our final simplification passing to the one-dimensional space  $0 < x < a$  and imposing the boundary conditions for two metallic plates

$$A(x=0) = A(x=a) = Y_\omega(x=0) = Y_\omega(x=a) = 0.$$

For the electromagnetic field this means that the plates are ideal reflectors, all dissipation coming only from the dielectric between the plates. As to the reservoir field  $Y$ , the boundary conditions may of course be taken in different ways but having in mind our restricted aim presented in the Introduction we choose the simplest and most convenient form.

Then the lagrangian becomes a sum over discrete values of momentum

$$k_n = \frac{\pi n}{a}, \quad n = 1, 2, \dots$$

and fields can be expanded as

$$A(x) = \sqrt{\frac{2}{a}} A_n \sin(k_n x), \quad Y_\omega(x) = \sqrt{\frac{2}{a}} Y_{n\omega} \sin(k_n x).$$

Quantization then follows in the standard manner, introducing the conjugate fields,  $\pi(x)$  and  $\Pi_\omega(x)$  for  $A(x)$  and  $Y_\omega(x)$  respectively, and imposing the standard commutation relations. In terms of creation and annihilation operators  $a_n, a_n^\dagger$  for the electromagnetic field and  $b_{n,\omega}, b_{n,\omega}^\dagger$  for the medium

$$A_n = \frac{1}{\sqrt{2\omega_n}} (a_n + a_n^\dagger), \quad \pi_n = -i \sqrt{\frac{\omega_n}{2}} (a_n - a_n^\dagger), \quad (7)$$

and

$$Y_{n,\omega} = \frac{i}{\sqrt{2\omega_n}} (b_{n,\omega} - b_{n,\omega}^\dagger), \quad \Pi_{n,\omega} = \sqrt{\frac{\omega}{2}} (b_{n,\omega} + b_{n,\omega}^\dagger). \quad (8)$$

One finds two equivalent expressions for the total Hamiltonian. In terms of fields and their time derivatives

$$H = \frac{1}{2} \sum_n \left\{ \dot{A}_n^2 + k_n^2 A_n^2 + \int_0^\infty d\omega (\dot{Y}_{n,\omega}^2 + \omega^2 Y_{n,\omega}^2) \right\} \quad (9)$$

or in terms of fields and their conjugates

$$H = \frac{1}{2} \sum_n \left( \pi_n^2 + k_{1n}^2 A_n^2 + \int_0^\infty d\omega (\Pi_{n,\omega}^2 + \omega^2 Y_{n,\omega}^2) + \sum_n \int_0^\infty d\omega v(\omega) A_n \Pi_{n,\omega} \right), \quad (10)$$

where

$$k_{1n}^2 \equiv k_n^2 + \mu^2 = k_n^2 + \int_0^\infty d\omega v^2(\omega). \quad (11)$$

### 3 Fano diagonalization, ground state energy and Casimir energy

In terms of creation and annihilation operators the Hamiltonian has the form

$$H = \frac{1}{2} \sum_n \left[ k_{1n} \{a_n^\dagger, a_n\} + \int_0^\infty d\omega \omega \{b_{n\omega}^\dagger, b_{n\omega}\} + \frac{1}{2} \int_0^\infty d\omega (\omega) [a_n^\dagger + a_n] [V_n(\omega) b_{n\omega}^\dagger + V_n^*(\omega) b_{n\omega}] \right]. \quad (12)$$

Here

$$V_n(\omega) = \sqrt{\frac{\omega}{k_{1n}}} v(\omega).$$

It can be demonstrated that the Hamiltonian  $H$  can be diagonalized introducing new field variables [1]

$$B_{n,\omega} = \alpha_{0n}(\omega) a_n + \beta_{0n}(\omega) + \int_0^\infty d\omega' \left[ \alpha_{1n}(\omega, \omega') b_{n,\omega'} + \beta_{1n}(\omega, \omega') b_{n,\omega'}^\dagger \right], \quad (13)$$

which satisfy the commutation relations

$$[B_{n,\omega}, B_{n',\omega'}^\dagger] = \delta_{nn'} \delta(\omega - \omega'), \quad [B_{n,\omega}, B_{n',\omega}] = 0. \quad (14)$$

These commutation relations together with the requirement that

$$[B_{n,\omega}, H] = \omega B_{n,\omega} \quad (15)$$

uniquely define the coefficients  $\alpha_{0n}$ ,  $\beta_{0n}$ ,  $\alpha_{1n}$ ,  $\beta_{1n}$  (see [1]):

$$\alpha_0(\omega) = -\left(\frac{\omega + \omega_1}{2}\right) V_n(\omega) P_n^*(\omega), \quad \beta_0(\omega) = -\left(\frac{\omega - \omega_1}{2}\right) V_n(\omega) P_n^*(\omega), \quad (16)$$

$$\alpha_1(\omega, \omega') = \delta(\omega - \omega') - \left(\frac{\omega_1}{2}\right) \left(\frac{V_n^*(\omega') V_n(\omega)}{\omega - \omega' - i0}\right) P_n^*(\omega), \quad (17)$$

$$\beta_1(\omega, \omega') = -\left(\frac{\omega_1}{2}\right) \left(\frac{V_n(\omega') V_n(\omega)}{\omega + \omega'}\right) P_n^*(\omega), \quad (18)$$

where

$$P_n(\omega) = \frac{1}{k^2 - \epsilon(\omega) \omega^2} \quad (19)$$

is the propagator of the electromagnetic field in the medium and

$$\epsilon(\omega) = 1 + \frac{k_{1n}}{\omega} \int_{-\infty}^{\infty} \frac{d\omega'}{\omega'} \frac{|V_n(\omega')|^2}{\omega' - \omega - i0} \quad (20)$$

is the dielectric constant.

In terms of the new variables Hamiltonian  $H$  has the form

$$H = \sum_n \int_0^{\infty} d\omega \omega B_{n,\omega}^\dagger B_{n,\omega} + E(a). \quad (21)$$

Here  $E(a)$  is a constant which obviously has the meaning of the ground state energy of the system.

Assuming that the new operators  $B_{n,\omega}$  and  $B_{n,\omega}^\dagger$  form a complete set one can invert relations (13) and its conjugate and express the initial operators as linear superpositions of the new ones. Comparing the commutation relations between the old and new operators written in terms of the old and new operators one obtains

$$\begin{aligned} a_n &= \int_0^{\infty} d\omega \left[ \alpha_{0n}^*(\omega) B_{n,\omega} - \beta_{0n}(\omega) B_{n,\omega}^\dagger \right], \\ b_{n,\omega} &= \int_0^{\infty} d\omega' \left[ \alpha_{1n}^*(\omega', \omega) B_{n,\omega'} - \beta_{1n}(\omega', \omega) B_{n,\omega'}^\dagger \right]. \end{aligned} \quad (22)$$

This procedure is consistent provided a certain consistency relation is satisfied [1]:

$$\int_0^{\infty} d\omega v |\omega|^2 < k_{1n}, \quad (23)$$

which in our case is true due to (11).

Expressing in the expression for the Hamiltonian the old operators  $A_n$ ,  $Y_{n,\omega}$  their conjugates and time derivatives via the new ones and taking the average in the ground state determined by the condition  $B_{n,\omega}|0\rangle = 0$  one can find the ground state energy  $E_0$ . One can use both forms (9) and (10) for this purpose. With (9) used in [7] the resulting formulas are simpler and directly expressed via the dielectric constant  $\epsilon(\omega)$ . However, as a price, in the course of the derivation one has to disentangle finite contributions from initially singular expressions. Adjusting the results of [7] to our one-dimensional case one finds the ground state energy

$$E_0 = \frac{1}{2\pi} \sum_n \int_0^{\infty} d\omega \operatorname{Im} \left( \omega^2 \frac{d}{d\omega} [\omega \epsilon(\omega)] + k_n^2 \right) P_n(\omega). \quad (24)$$

For calculations one standardly rotates the contour to pass along the positive imaginary axis to find

$$E_0 = \frac{1}{2\pi} \sum_n \int_0^{\infty} d\xi \left\{ k_n^2 - \xi^2 \left( \frac{d}{d\omega} [\omega \epsilon(\omega)] \right)_{\omega=i\xi} \right\} P_n(i\xi) \quad (25)$$

where the integrand is real.

To find the Casimir energy one has to subtract from  $E_0$  its value for the case when there are no plates, that is for  $k$  continuously distributed in the interval  $[0, \infty)$

$$E_{cas} = E_0(a) - \tilde{E}_0(a).$$

In  $\tilde{E}_0$  the summation over  $k_n$  is changed to integration over  $k$  with weight  $a/\pi$ , which implies in our formulas

## 4 Simple generalizations.

In this section to study the dependence of the Casimir energy and force on the assumed model for the dispersive and absorbing medium we study simple generalizations of the model presented before. From the start to simplify we restrict ourselves to the same picture of one dimensional fields between two metallic plates.

### 4.1 Interaction with several $\dot{Y}$

The absorbing medium is now modeled by a set of different oscillators  $Y_{j,\omega}(x)$  in the same interval of coordinates  $[0, a]$  with continuously distributed frequencies:

$$L_1 = \frac{1}{2} \int_0^a dx \int_0^\infty d\omega \sum_{j=1}^N \left( \dot{Y}_j^2 - \omega^2 Y_j^2 \right). \quad (26)$$

The interaction between the quantum field and the medium can be generalized as

$$L_I = - \int_0^a dx \int_0^\infty d\omega A \sum_{j=1}^N v_j \dot{Y}_j, \quad (27)$$

where  $v^2(j, \omega)$  are square integrable functions which can be analitically continued to negative  $\omega$  as an even function.

We make a unitary transformation to new variables  $Y'_{j,\omega}(x)$

$$Y'_{j,\omega}(x) = \sum_{l=1}^N u_{jl}(\omega) Y_{l,\omega}(x) \quad (28)$$

and take

$$\sum_{l=1}^N v_l(\omega) Y_{l,\omega}(x) = v(\omega) Y'_{1,\omega}(x), \quad v_l(\omega) = v(\omega) u_{1l}(\omega). \quad (29)$$

From the unitarity of the transformation we have

$$v(\omega) = \sqrt{\sum_{l=1}^N v_l^2(\omega)}. \quad (30)$$

In terms of new variables  $L_1$  does not change but  $L_I$  becomes dependent only on  $Y'_1$

$$L_I = - \int_0^a dx \int d\omega v(\omega) A \dot{Y}'_{1,\omega}, \quad (31)$$

As a result the model is completely equivalent to the one with a single  $Y$ , all the additional variables of the medium not interacting with the electromagnetic field.

### 4.2 Interaction with both $\dot{Y}$ and $Y$

Next we study a generalization to interaction with both  $\dot{Y}$  and  $Y$  with the interaction Lagrangian

$$L_I = - \int_0^a dx \int d\omega A \left( v_1 \dot{Y} - v_2 Y \right), \quad (32)$$

To quantize we determine the conjugated variables as usual and find the Hamiltonian in the form  $H_e + H_Y + H_I$  where  $H_e$  and  $H_Y$  for the free electromagnetic field and medium are the same as before but with the mass shift

$$\mu^2 = \int_0^\infty d\omega v_1^2(\omega). \quad (33)$$

Remarkably it depends only on  $v_1$ . The interaction is now

$$H_I = \int_0^a dx \int_0^\infty d\omega A \left( v_1 \Pi_\omega + v_2 Y_\omega \right). \quad (34)$$

Our strategy is the same: we try to reduce this to the old model. First we rescale variables  $Y$  and  $\Pi$

$$Y_\omega = \frac{1}{\sqrt{\omega}} Q_\omega, \quad \Pi_\omega = \sqrt{\omega} P_\omega, \quad (35)$$

so that

$$H_1 = \int_0^a dx \int_0^\infty d\omega \omega (Q_\omega^2 + P_\omega^2). \quad (36)$$

Next we do a canonical transformation

$$Q'_\omega = Q_\omega \cos \theta + P_\omega \sin \theta, \quad P'_\omega = -Q_\omega \sin \theta + P_\omega \cos \theta. \quad (37)$$

It preserves the form of  $H_1$  and commutation relations between  $Q$  and  $P$  which are standard. Now we identify

$$v_1 \Pi_\omega + v_2 Y_\omega = \frac{v_2}{\sqrt{\omega}} Q_\omega + v_1 \sqrt{\omega} P_\omega = \tilde{v}(\omega) P'_\omega = \tilde{v}(\omega) \left( -Q_\omega \sin \theta + P_\omega \cos \theta \right). \quad (38)$$

Comparison gives

$$\tilde{v}(\omega) \sin \theta = -\frac{v_2}{\sqrt{\omega}}, \quad \tilde{v}(\omega) \cos \theta = v_1 \sqrt{\omega}, \quad (39)$$

so that

$$\tilde{v}^2 = \omega v_1^2 + \frac{1}{\omega} v_2^2. \quad (40)$$

Returning to the natural momenta

$$P'_\omega = \frac{1}{\sqrt{\omega}} \Pi'_\omega, \quad Q'_\omega = \sqrt{\omega} Y_\omega \quad (41)$$

we find that  $H_1$  is the same but the interaction is now

$$H_I = \int_0^a dx \int_0^\infty d\omega v(\omega) A \Pi'_\omega, \quad (42)$$

where

$$v^2 = \frac{1}{\omega} \tilde{v}^2 = v_1^2 + \frac{v_2^2}{\omega^2}. \quad (43)$$

We see a problem: the mass shift depends on only  $v_1$ . As a result the necessary conditions for the consistency of the quantization [1] become violated and the propagator develops an extra pole on the imaginary axis, which violates validity of the commutation relations

To remedy this defect one has to include an extra term into the interaction to make the mass shift consistent with the interaction  $H_I$ :

$$\Delta H_I = -\Delta L_I = \frac{1}{2} \int_0^a dx A^2 \int_0^\infty \frac{d\omega}{\omega^2} v_2^2(\omega). \quad (44)$$

The new mass-shift added to the right-hand side of Eq. (11) converts the final mass into

$$\mu^2 = \int_0^\infty d\omega v^2(\omega), \quad (45)$$

### 4.3 General case

Now we are in the position to consider the case of  $N$  oscillators in the medium with a general interaction with the field

$$H_I = \int_0^a dx \int_0^\infty d\omega A \sum_{j=1}^N \left( v_j^{(1)} \Pi_j + v_j^{(2)} Y_j \right) \quad (46)$$

with the mass shift

$$\mu^2 = \sum_{j=1}^N v_j^{(1)2}. \quad (47)$$

We first canonically transform each pair  $Y_j, \Pi_j$  to  $Y'_j, \Pi'_j$  as before to reduce the interaction to

$$H_I = \int_0^a dx \int_0^\infty d\omega A \sum_{j=1}^N v_j^{(0)} \Pi'_j, \quad (48)$$

where

$$v_j^{(0)2} = v_j^{(1)2} + \frac{v_j^{(2)2}}{\omega^2}. \quad (49)$$

Then we act as in the first subsection and unitary transform  $Y'_j$  and  $\Pi'_j$  between themselves to reduce the interaction to only with the  $\Pi''_1$

$$H_I = \int_0^a dx \int_0^\infty d\omega A \Pi''_1 \quad (50)$$

where now

$$v^2 = \sum_{j=1}^N v_j^{(0)2}. \quad (51)$$

To secure absence of the poles of the propagator on the complex plane we additionally introduce extra interaction in the form

$$\Delta H_I = -\Delta L_I = \frac{1}{2} \int_0^a dx A^2 \int_0^\infty \frac{d\omega}{\omega^2} \sum_j v_j^{(2)2}(\omega), \quad (52)$$

so that the final mass turns out into

$$\mu^2 = \int_0^\infty d\omega v^2(\omega) \quad (53)$$

in accordance with the necessary conditions for the absence of extra poles. As a result, effectively this generalized model is equivalent to the old one.

## 5 Further generalization: two stage scenario (the HB model)

### 5.1 The model

In this section we introduce the picture of the medium originally proposed by T.Huttner and S.M.Barnet in [1] in which the electromagnetic field interacts with the absorbing medium not directly but via an oscillator imitating the atom immersed in the medium. The transverse Lagrangian density instead of Eq. (5) is taken to be



$$-\alpha \mathbf{A} \cdot \dot{\mathbf{X}}_{\perp} - \int_0^{\infty} d\omega v(\omega) \mathbf{X}_{\perp} \cdot \dot{\mathbf{Y}}_{\omega\perp}. \quad (54)$$

with a new field  $\mathbf{X}$  representing the atom. Correspondingly to our expressions for the energy new terms are to be added corresponding to the free field  $\mathbf{X}$  and its interaction with the electromagnetic field and medium. Thus instead of Eqs. (9) and (10) in the one-dimensional case we find two equivalent expressions for the new Hamiltonian

$$H = \frac{1}{2} \sum_n \left( \dot{A}_n^2 + k_n^2 A_n^2 + \dot{X}_n^2 + \omega_0^2 X_n^2 \right) + \int_0^{\infty} d\omega (\dot{Y}_{n,\omega}^2 + \omega^2 Y_{n,\omega}^2) \quad (55)$$

or in terms of fields and their conjugates  $\pi$ ,  $q$  and  $\Pi_{\omega}$  for  $A$ ,  $X$  and  $Y_{\omega}$  respectively

$$H = \frac{1}{2} \sum_n \left( \pi_n^2 + k_{1n}^2 A_n^2 + q_n^2 + \omega_1 X_n^2 \right) + \int_0^{\infty} d\omega \left( \Pi_{n,\omega}^2 + \omega^2 Y_{n,\omega}^2 \right) \\ + \sum_n \left( -\alpha A_n \dot{X}_n - \int_0^{\infty} d\omega v(\omega) X_n \Pi_{n,\omega} \right), \quad (56)$$

where now

$$k_{1n}^2 = k_n^2 + \alpha^2, \quad \omega_1^2 = \omega_0^2 + \int_0^{\infty} d\omega v^2(\omega). \quad (57)$$

For the following we shall need the expression for the Hamiltonian in terms of annihilation and creation operators  $a_n, a_n^{\dagger}$ ,  $b_n, b_n^{\dagger}$  and  $b_{n\omega}, b_{n\omega}^{\dagger}$  for the fields  $A, X$  and  $Y_{\omega}$  respectively. We have

$$H = \tilde{H} + E_{e0} + E_{X0} + E_{Y0} \quad (58)$$

where

$$\tilde{H} = H_e + H_X + H_Y + H_{XY} + H_{eX} \quad (59)$$

with the free parts

$$H_e = \sum_n k_{1n} a_n^{\dagger} a_n, \quad H_x = \sum_n \omega_1 b_n^{\dagger} b_n, \quad H_Y = \sum_n \int_0^{\infty} d\omega \omega b_{n,\omega}^{\dagger} b_{n,\omega} \quad (60)$$

and the interaction parts

$$H_{XY} = \frac{1}{2} \int_0^{\infty} d\omega V(\omega) [b_n^{\dagger} + b_n] [b_{n,\omega}^{\dagger} + b_{n,\omega}], \quad H_{eX} = \frac{1}{2} i \sum_n \Lambda_n [a_n^{\dagger} + a_n] [b_n^{\dagger} - b_n]. \quad (61)$$

Here

$$V^2(\omega) = v^2(\omega) \frac{\omega}{\omega_1}, \quad \Lambda_n^2 = \alpha^2 \frac{\omega_1}{k_{1n}}, \quad k_{1n}^2 = k^2 + \alpha^2 \quad (62)$$

and

$$E_{e0} = \frac{1}{2} \sum_n k_{1n}, \quad E_{X0} = \frac{1}{2} \sum_n \omega_1, \quad E_{Y0} = \frac{1}{2} \sum_n \int_0^{\infty} d\omega \omega. \quad (63)$$

## 5.2 Two-stage Fano diagonalization

In the approach of HB one first Fano-diagonalizes the "matter" Hamiltonian  $H_Y + H_{XY}$  with a real function  $V(\omega)$ . The consistency relation is automatically satisfied, since

$$\int_0^{\infty} \frac{d\omega}{\omega} V^2(\omega) = \frac{1}{\omega_1} \int_0^{\infty} d\omega v^2(\omega) < \omega_1, \quad (64)$$

which is fulfilled due to the definition of  $\omega_1$ , Eq. (57). After this first step the total Hamiltonian acquires the form

$$H_1 = \frac{1}{2} \sum_n \left[ k_{1n} \{a_n^\dagger, a_n\} + \int_0^\infty d\omega \omega \{B_{n\omega}^\dagger, B_{n\omega}\} + i\Lambda_n [a_n^\dagger + a_n] [b_n^\dagger - b_n] \right]. \quad (65)$$

To transform it to the standard form (12) we have to express  $b$  in terms of  $B_\omega$ :

$$b_n = \int_0^\infty d\omega \left[ \alpha_0^*(\omega) B_{n\omega} - \beta_0(\omega) B_{n\omega}^\dagger \right] \quad (66)$$

and its complex conjugate. So

$$b_n^\dagger - b_n = \int_0^\infty d\omega \left[ B_{n\omega}^\dagger \left( \alpha_0(\omega) + \beta_0(\omega) \right) - c.c \right]$$

and the Hamiltonian  $H_1$  takes the form (12) with

$$V_n(\omega) \rightarrow V_{1n}(\omega) = i\Lambda_n \left( \alpha_0(\omega) + \beta_0(\omega) \right) = -i\Lambda_n \omega V(\omega) Q^*(\omega), \quad (67)$$

where  $Q(\omega)$  is the "propagator" for the field  $X$  in the medium

$$Q(\omega) = \frac{1}{\omega_0^2 - \omega^2 \sigma(\omega)}$$

and  $\sigma$  playing the role of the "dielectric constant"

$$\sigma(\omega) = 1 + \frac{\omega_1}{2\omega} \int_0^\infty d\omega' \frac{d\omega'}{\omega'} \frac{|V(\omega')|^2}{\omega' - \omega - i0}.$$

Note that  $k_{1n}|V_1(k, \omega)|^2$  does not depend on  $k_{1n}$ . One can check that

$$\int_0^\infty d\omega \frac{d\omega}{\omega} |V_{1n}(\omega)|^2 = \frac{\alpha^2}{k_{1n}} < k_{1n}$$

and the consistency condition for the second diagonalization is fulfilled.

To finally diagonalize the Hamiltonian we define operators

$$C_{n\omega} = \xi_{0n}(\omega) a_n + \eta_{0n}(\omega) a_n^\dagger + \int d\omega' \left( \xi_{1n}(\omega, \omega') B_{n\omega'} + \eta_{1n}(\omega, \omega') B_{n\omega'}^\dagger \right). \quad (68)$$

The resulting coefficients are found to be

$$\xi_{0n} = -\left( \frac{\omega + k_{1n}}{2} \right) V_{1n}(\omega) P_n^*(\omega), \quad \eta_{0n}(\omega) = -\left( \frac{\omega - k_{1n}}{2} \right) V_1(\omega) P_n^*(\omega) \quad (69)$$

and

$$\xi_{1n}(\omega, \omega') = \delta(\omega - \omega') - \frac{k_{1n}}{2} P_n^*(\omega) \frac{V_{1n}^*(\omega') V_{1n}(\omega)}{\omega - \omega' - i0}, \quad (70)$$

$$\eta_{1n}(\omega, \omega') = -\frac{k_{1n}}{2} P_n^*(\omega) \frac{V_{1n}(\omega') V_{1n}(\omega)}{\omega + \omega'}. \quad (71)$$

Here  $P_n(\omega)$  is the propagator of the electromagnetic field (19) with the dielectric constant given by (20) with the substitution  $V \rightarrow V_1$ .

(as mentioned it does not depend on  $k$ ).

The initial operators can be expressed via  $C_\omega(\mathbf{k})$ . Similarly to (22) we have

$$a_n = \int_0^\infty d\omega \left[ \xi_{0n}^*(\omega) C_{n\omega} - \eta_{0n}(\omega) C_{n\omega}^\dagger \right] \quad (73)$$

and expressing  $B$  via  $C$  we find

$$b_n = \int_0^\infty d\omega \left( \mu_{0n}^*(k, \omega) C_{n\omega} - \nu_{0n}(\omega) C_{n\omega}^\dagger \right), \quad (74)$$

where

$$\mu_{0n}(\omega) = \int_0^\infty d\omega' \left( \alpha_{0n}(\omega') \xi_{1n}(\omega, \omega') + \beta_{0n}^*(\omega') \eta_{1n}(\omega, \omega') \right), \quad (75)$$

$$\nu_{0n}(\omega) = \int_0^\infty d\omega' \left( \alpha_{0n}^*(\omega') \eta_{1n}(\omega, \omega') + \beta_{0n}(\omega') \xi_{1n}(\omega, \omega') \right) \quad (76)$$

and furthermore

$$b_{n\omega} = \int_0^\infty d\omega' \left( \mu_{1n}^*(\omega', \omega) C_{n\omega'} - \nu_{1n}(\omega', \omega) C_{n\omega'}^\dagger \right), \quad (77)$$

where

$$\mu_{1n}(\omega', \omega) = \int_0^\infty d\omega'' \left( \alpha_1(\omega'', \omega) \xi_{1n}(\omega', \omega'') + \beta_1^*(\omega'', \omega) \eta_{1n}(\omega', \omega'') \right), \quad (78)$$

$$\nu_{1n}(\omega', \omega) = \int_0^\infty d\omega'' \left( \alpha_1^*(\omega'', \omega) \eta_{1n}(\omega', \omega'') + \beta_1(\omega'', \omega) \xi_{1n}(\omega', \omega'') \right). \quad (79)$$

### 5.3 The ground state energy

Expressing operators  $a$ ,  $b$  and  $b_\omega$  via  $C_\omega$  according to Eqs. (73), (74) and (77) and averaging in the ground state with  $C_{n\omega}|0\rangle = 0$  find

$$\langle H_e \rangle = \sum_n k_{1n} \int_0^\infty d\omega |\eta_{0n}|^2, \quad (80)$$

$$\langle H_X \rangle = \sum_n \int_0^\infty d\omega \omega |\nu_{0n}(\omega)|^2, \quad (81)$$

$$\langle H_Y \rangle = \sum_n \int_0^\infty d\omega \omega \int_0^\infty d\omega' |\nu_{1n}(\omega', \omega)|^2. \quad (82)$$

Passing to the interaction terms

$$\langle H_{XY} \rangle = \frac{1}{2} \sum_n \int_0^\infty d\omega V(\omega) \int_0^\infty d\omega' \left( \mu_{0n}^*(\omega') - \nu_{0n}^*(\omega') \right) \left( \mu_{1n}(\omega', \omega) - \nu_{0n}(\omega', \omega) \right) \quad (83)$$

and

$$\langle H_{eX} \rangle = \frac{1}{2} i \sum_n \Lambda_n \int_0^\infty d\omega \left( \xi_{0n}^*(\omega) - \eta_{0n}^*(\omega) \right) \left( \mu_{0n}(\omega) + \nu_{0n}(\omega) \right). \quad (84)$$

Using relations between the coefficients we can rewrite the two contributions from the interaction as

$$\langle H_{XY} \rangle = \frac{1}{2} \sum_n \int_0^\infty d\omega V(\omega) \int_0^\infty d\omega' \left[ \nu_{1n}^*(\omega', \omega) \left( \nu_{0n}(\omega') - \mu_{0n}(\omega') \right) + c.c. \right] \quad (85)$$

and

$$\langle H_{eX} \rangle = -\frac{1}{2} i \sum_n \Lambda_n \int_0^\infty d\omega \left[ \eta_{0n}^*(\omega) \left( \mu_{0n}(\omega) + \nu_{0n}(\omega) \right) - c.c. \right]. \quad (86)$$

## 6 Comparison of models with direct and indirect interaction with the medium

### 6.1 Generalities

Our central aim is to study whether the Casimir energy depends only on the dielectric constant  $\epsilon(\omega)$  and in this manner is independent of the model assumed for absorption or it depends on this model so that with the same  $\epsilon(\omega)$  one gets different results for different models for absorption. We have seen that in the simple model studied in Sections 2. and 3. with the direct interaction (D model) with the absorbing medium the Casimir energy is expressed entirely by  $\epsilon(\omega)$ . The same is true for its simple generalizations considered in Section 4. It remains to study the model introduced in Sections 5. – 7. in which the electromagnetic field interacts indirectly with the medium, via the atomic oscillator immersed in the medium (HB model). Expression for the Casimir energy in the latter model are far from being transparent and their inspection does not allow to understand if the Casimir energy as before depends entirely on the dielectric constant or it is not true. So we recur to numerical check. We select parameters of the two models to give the same dielectric constant and then calculate the resulting Casimir energies.

A few words on the technique of the calculation. The normalization of the Casimir energy, as mentioned, requires subtracting the energy in absence of metallic plates, which implies calculating the difference

$$\sum_n - \int_0^\infty dn,$$

where in both terms  $k_n = \pi n/a$ . In this subtraction all terms independent of  $k_n$  do not contribute. Both the sum and the integral contain integration over frequency  $\omega$ . In the electromagnetic part there are terms divergent at  $\omega \rightarrow \infty$ . To ensure convergence it is convenient to separate from this part its value in absence of the dielectric, which is the standard Casimir energy.

$$E_{cas}^{(0)} = -\frac{\pi}{24a}.$$

Then after rotation to imaginary frequencies the energy in model D changes to

$$E_0 = \frac{1}{2\pi} \sum_n \int_0^\infty d\xi \left\{ \left( k_n^2 - \xi^2 \right) \left( P_n(i\xi) - P_n^{(0)}(i\xi) \right) - \xi^2 \left( \frac{d}{d\omega} [\omega(\epsilon(\omega) - 1)] \right)_{\omega=i\xi} P_n(i\xi) \right\}. \quad (87)$$

where  $P_n^{(0)}(\omega) = 1/(k_n^2 - \omega^2 - i0)$  is the free propagator. In model HB only the electromagnetic part is transformed into

$$\langle H_e \rangle_I = \frac{1}{2} \sum_n (k_{1n} - k_n) + \frac{1}{2\pi} \sum_n \int_0^\infty d\xi \left[ (k_n^2 - \xi^2) \left( P_n(i\xi) - P_n^{(0)}(i\xi) \right) + \alpha^2 P_n(i\xi) \right]. \quad (88)$$

The first term appears because  $E_{e0}$  in (63) depends on the interaction.

### 6.2 Numerical calculations

Integration over frequencies  $\omega$  cannot be efficiently done without rotating the contour to pass along the imaginary axis, since the propagator has a resonant behavior on the real axis. This rotation is trivial in model D and for the electromagnetic part in model HB but not so trivial for other terms in model HB, which contain factors with singularities in the first quadrant of the complex  $\omega$ -plane. In fact both the propagator and the dielectric constant are regular in the upper half plane, as it should be. But expression for the energy also depend on the complex conjugated

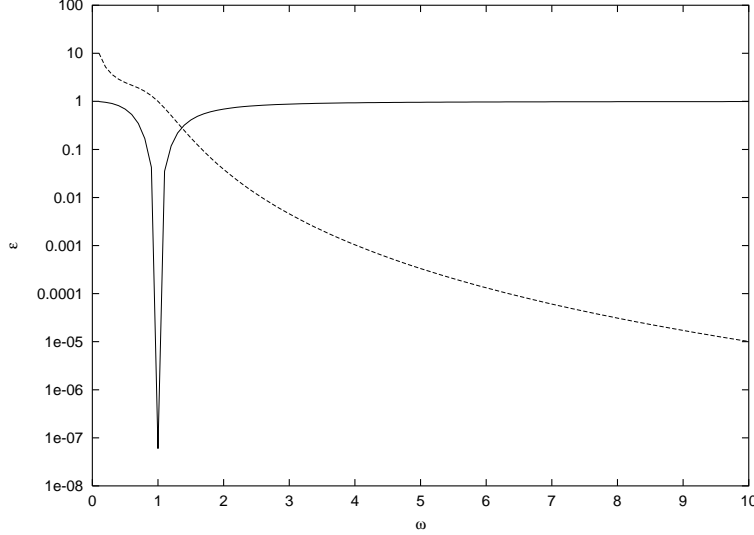


Figure 1: Real(upper curve for large  $\omega$ ) and imaginary parts of  $\epsilon(\omega) - 1$

upper half plane. Also in any case one would like to have analytic expressions for all terms in both models which allow to do the analytic continuation constructively. So we have chosen a particular form of the interaction  $v(\omega)$  in model HB which allows to find all expressions including the dielectric constant  $\epsilon(\omega)$  in the analytic form. This allowed to constructively do the continuation to imaginary frequencies in both models.

Our choice is

$$v(\omega) = \frac{g^2}{\omega^2 + m^2}, \quad g^2 = \frac{1}{2\pi}, \quad \omega_0 = 0. \quad (89)$$

With  $m$  the only dimensionful parameter, the additional Casimir energy due to the interaction with the dielectric is

$$E_{cas}^{(1)}(a) = m e_{cas}^{(1)}(ma).$$

In the following we put  $m = 1$ . With choice (89) we find the dielectric constant

$$\epsilon(\omega) = 1 - \alpha^2 \frac{\omega + i}{\omega(\omega^2 - 1 + i\omega)} \quad (90)$$

We do the calculation for the interaction parameter  $\alpha = 1$ . For illustration we show real and imaginary parts of  $\epsilon(\omega) - 1$  in this case in Fig. 1. In model HB we also meet quantities  $k_{1n}|V_{1n}(\omega)|^2 = \alpha^2 g^2 \omega / (\omega^4 - \omega^2 + 1)$  and  $\epsilon^*(\omega)$  which have a pole in the first quadrant at  $\omega = \omega_P = e^{1\pi/6}$ . As mentioned they come from complex conjugated terms, which appear in the expressions for the energy, in contrast to the D model. So in analytic continuation one had to take into account residues at this pole.

Our results for  $E_{cas}^{(1)}(a)$  and the force  $F_{cas}^{(1)}(a)$  are shown in Figs. 2 and 3 respectively.

Inspection of them clearly shows that the Casimir energy and force do depend on the way the electromagnetic field interacts with the absorbing medium. With indirect interaction both turn out to be considerably larger. In model D at large  $a \gg 1$  the additional energy tends to a finite negative value  $E_{cas}^{(1)}(a) \rightarrow -0.1618$  and the force falls exponentially  $F_{cas}^{(1)}(a) \sim \exp(-0.055a)$ . In contrast in model IN  $E_{cas}^{(1)}(a) \sim -8 \ln a$  and the force falls quite slowly  $F_{cas}^{(1)}(a) \sim 6/a$ . As to the absolute values of the additional contribution from the dielectric, they depend on the values chosen for the parameters  $\alpha$  and  $m$ . So the curve describing the vacuum energy in Fig. 2 serves

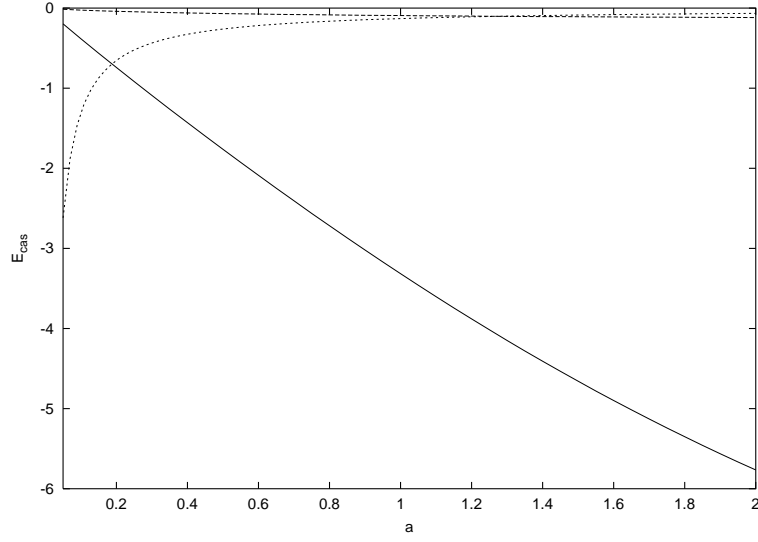


Figure 2: The additional Casimir energy in the dielectric in Models D (middle curve at large  $a$ ) and HB (lower curve at large  $\omega$ ). The upper curve at large  $\omega$  shows the Casimir energy in the vacuum

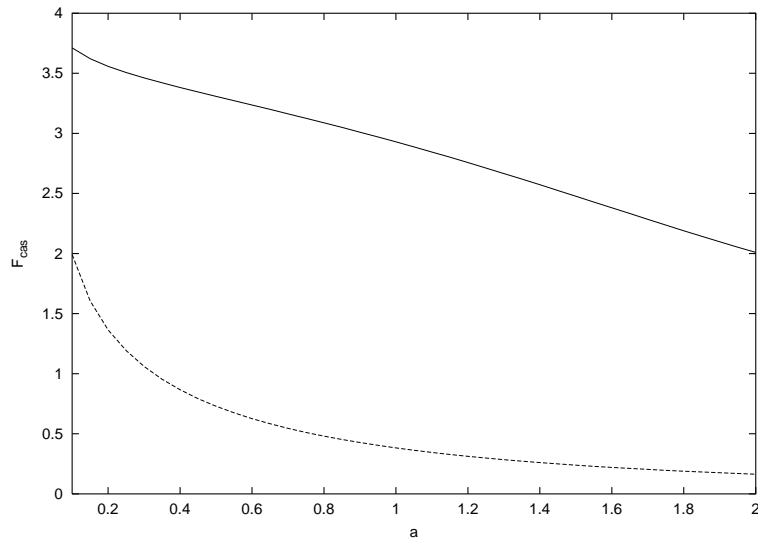


Figure 3: The additional Casimir force in the dielectric in Models D (multiplied by 10, lower curve) and HB (upper curve)

## 7 Conclusions

A consistent quantization of the electromagnetic field in the dielectric in the microscopic treatment remains an important problem with far-reaching impact. It should give the answer to the macroscopic treatment of the same problem, in which all influence of the medium is entirely contained in the complex dielectric constant. If this is indeed so then different microscopic models for dispersion and absorption leading to the same dielectric constant should give the same Casimir energy. Our study of two essentially different models for the interaction with the medium, one in which the electromagnetic field interacts directly with the absorbing medium and the other via the intermediary (atom) has shown that the answer is negative. With the same dielectric constant the models give different Casimir energies and forces. This means that the interaction with the dielectric in the microscopic approach is not entirely encoded in the dielectric constant, so that the Casimir energy has an extra dependence on the interaction with the dielectric, which, in principle, can be studied experimentally.

Our conclusions are based on the study of very particular (although well-known) microscopic models of absorption and moreover on a specific parametrization of the interaction. However this is sufficient to demonstrate the above-mentioned conclusion. In fact our formulas, in principle, allow to calculate the Casimir energy for arbitrary forms of interaction within the studied models. However realistic calculations in model HB with indirect interaction with the medium seem rather difficult. So at present we cannot find the Casimir energy in model HB with different forms of interaction and so check whether the found energy depends on these forms or is a characteristic of the model as such.

## 8 Acknowledgments

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## 9 Comments on the final formulas for the calculation

There exist certain relations between the coefficients introduced in Section 5.2 which guarantee fulfilment of commutation relations between initial operators. Expressing

$$[a_n, b_{n'}] = [a_n, b_{n'}^\dagger] = 0 \quad (91)$$

in terms of  $C_\omega$  one gets

$$\int_0^\infty d\omega \left( -\xi_{0n}^*(\omega) \nu_{0n}(\omega) + \eta_{0n}(\omega) \mu_{0n}^*(\omega) \right) = 0$$

and

$$\int_0^\infty d\omega \left( \xi_{0n}^*(\omega) \mu_{0n}(\omega) - \eta_{0n}(\omega) \nu_{0n}^*(\omega) \right) = 0.$$

Similarly from

$$[b_n, b_{n'\omega}] = [b_n, b_{n'\omega}^\dagger] = 0 \quad (92)$$

one finds

$$\int_0^\infty d\omega' \left( -\mu_{0n}^*(\omega') \nu_{1n}(\omega', \omega) + \nu_{0n}(\omega') \mu_{1n}^*(\omega', \omega) \right) = 0$$

and

These relations can be used to simplify some expressions for the ground state energy.

For particular cotributions to the energy we have the following comments.

**1.  $\langle H_e \rangle$**

Using (69) we find

$$|\eta_{0n}(\omega)|^2 = \frac{1}{4}(\omega - k_{1n})^2 |V_{1n}|^2 |P_n(\omega)|^2. \quad (93)$$

Since

$$\text{Im } P_n(\omega) = \omega^2 \text{Im } \epsilon(\omega) |P_n(\omega)|^2 = \pi \frac{k_{1n}}{2} |V_{1n}|^2 |P_n(\omega)|^2,$$

we obtain

$$\langle H_e \rangle = \frac{1}{2\pi} \text{Im} \sum_n \int_0^\infty d\omega (\omega - k_{1n})^2 P_n(\omega). \quad (94)$$

The integral admits rotation to the positive imaginary axis.

**2.  $\langle H_X \rangle$**

Coefficient  $\nu_{0n}$  is given by Eq. (76) with  $\alpha_0$  and  $\beta_0$  are given by (16). and coefficients  $\xi_{1n}(\omega, \omega')$  and  $\eta_{1n}(\omega, \omega')$  are given by formulas (70) and (71) So

$$\nu_{0n} = \left( \frac{\omega - \omega_1}{2} \right) \frac{V_{1n}(\omega)}{i\Lambda_n \omega} + i \frac{k_{1n}}{4\Lambda_n} P_n^*(\omega) V_{1n}(\omega) J_1(\omega),$$

where

$$J_1(\omega) = \int_0^\infty d\omega' \frac{|V_{1n}(\omega')|^2}{\omega'} \left( \frac{\omega' - \omega_1}{\omega - \omega' - i0} - \frac{\omega' + \omega_1}{\omega + \omega'} \right) = (\omega - \omega_1) J - 2 \int_0^\infty d\omega' \frac{|V_{1n}(\omega')|^2}{\omega} \quad (95)$$

with

$$\begin{aligned} J &= \int_0^\infty d\omega' \frac{|V_{1n}(\omega')|^2}{\omega'} \left( \frac{1}{\omega - \omega' - i0} + \frac{1}{\omega + \omega'} \right) = \int_{-\infty}^{+\infty} \frac{|V_{1n}(\omega')|^2}{\omega'} \frac{1}{\omega - \omega' - i0} \\ &= \frac{2\omega}{k_{1n}} \left( 1 - \epsilon^*(\omega) \right). \end{aligned} \quad (96)$$

The second integral in  $J_1$  is equal to  $\alpha^2/k_{1n}$  So

$$J_1 = \frac{2}{k_{1n}} \left[ \omega(\omega - \omega_1) \left( 1 - \epsilon^*(\omega) \right) - \alpha^2 \right]. \quad (97)$$

This leads to our final result

$$\nu_{0n}(\omega) = i \frac{V_{1n}(\omega)}{2\Lambda_n \omega} \left[ \omega_1 - \omega - (\omega_1 - \omega) P_n^*(\omega) \omega^2 \left( 1 - \epsilon^*(\omega) \right) - \alpha^2 \omega P_n^*(\omega) \right]. \quad (98)$$

**3.  $\langle H_Y \rangle$**

Coefficient  $\nu_{1n}$  is given by (79). Coefficients  $\alpha_1$  and  $\beta_1$  are given by (17) and (18). Coefficients  $\xi_{1n}$  and  $\eta_{1n}$  are given by (70) and (71). As a result we find  $\nu_{1n}$  as a sum of three terms

$$\begin{aligned} \nu_{1n}(\omega', \omega) &= -\frac{k_{1n}}{2} P_n^*(\omega') \frac{V_{1n}(\omega) V_{1n}(\omega')}{\omega' + \omega} - i \frac{\omega_1}{2\Lambda_n \omega'} \frac{V(\omega) V_{1n}(\omega')}{\omega' + \omega} \\ &\quad + i \frac{k_{1n}}{4\alpha} \sqrt{k_{1n} \omega_1} P_n^*(\omega') V(\omega) V_{1n}(\omega') J_2, \end{aligned} \quad (99)$$

where



$$\begin{aligned}
&= \frac{1}{\omega + \omega'} \int_0^\infty d\omega'' \frac{|V_{1n}(\omega'')|^2}{\omega''} \left( \frac{1}{\omega - \omega'' - i0} + \frac{1}{\omega + \omega''} + \frac{1}{\omega' - \omega'' - i0} + \frac{1}{\omega' + \omega''} \right) \\
&= \frac{1}{\omega + \omega'} \left( J(\omega) + J(\omega') \right) = \frac{2}{k_{1n}} \frac{\omega \left( 1 - \epsilon^*(\omega) \right) + \omega' \left( 1 - \epsilon^*(\omega') \right)}{\omega + \omega'}.
\end{aligned}$$

#### 4. $\langle H_{XY} \rangle$

Coefficients  $\nu_{0n}$  and  $\nu_{1n}$  are already given by (98) and (99). Coefficient  $\mu_{0n}$  differs from  $\nu_{0n}$  by the change  $\alpha_0 \leftrightarrow \beta_0$ , that is  $\omega_1 \rightarrow -\omega_1$ . So

$$\mu_{0n}(\omega) = i \frac{V_{1n}(\omega)}{2\Lambda_n \omega} \left[ -\omega_1 - \omega + (\omega_1 + \omega) P_n^*(\omega) \omega^2 \left( \epsilon(\omega) - 1 \right) - \alpha^2 \omega P_n^*(\omega) \right]. \quad (100)$$

#### 5. $\langle H_{eX} \rangle$

Here all coefficients are already known.

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